Exercise 3.1

**Question 2.**

A. True. Every integer is a real number. The set of real numbers ( R) includes all the integers ( Z), as well

as fractions, rational numbers, and irrational numbers.

B. False. Zero is not a positive real number; it is non-negative. Positive numbers are greater than zero.

C. True. For any real number r, −r is a negative real number if r is positive. If r is negative, then − −r is

positive. The statement does not consider the case when r is zero, in which case −r is also zero, not

negative.

D. True. Every real number is an integer or has a fractional part. An integer is a whole number and does

not have a fractional part.

Question 3.

1.A/ P(2): “2 > 1/2”. This is TRUE because 2 is greater than 0.5.

1.B/ P(1/2): “1/2 > 1/(1/2)” False because ½ is less than 2.

1.C/ P(-1): “

-1 >1/-1” False because -1 isn’t greater than -1.

1.D/ P(-1/2):”

-1/2 > 1/(-1/2). False because -1/2 isn’t greater than -2.

1.E/ P(-8): “

-8 > 1/-8” False because -8 isn’t greater than -0.125.

B. The truth set of P(x) in the domain of all real numbers is the set of all x such that x > 1/x. This

inequality holds for x > 1 and for negative x such that x < −1.

C. If the domain is the set R+ of all positive real numbers, the truth set of P(x) is the set of all positive

real numbers greater than 1 because the inequality x>1/x holds for all x > 1. It does not hold for 0 < x < 1

since in that interval 1 / x > x.

Question 4.

A. Q(2): This checks if 2^2 ≤30. Since 4 ≤ 30, this statement is true.

A. Q(−2): This checks if (−2)^2 ≤30. Since 4≤30, this statement is also true.

A. Q(7): This checks if 7^2 ≤30. Since 49 ≤30, this statement is false.

A. Q(−7): This checks if (−7)^2 ≤30. Since 49 ≤30, this statement is false.

B. The truth set of Q(n) when the domain is Z, the set of all integers, consists of all integers n such that

n^2 ≤30. The integers that satisfy this are − 5 , − 4 , − 3 , − 2 , − 1 , 0 , 1 , 2 , 3 , 4 , 5 since 5^2 =25 is the

largest square less than or equal to 30.

C. If the domain is the set Z+ of all positive integers, the truth set of Q(n) is the set of all positive integers

n such that n^2 ≤30. The positive integers that satisfy this are 1 , 2 , 3 , 4 , 5.

Question 5.

a. Explain why (Q(x, y)) is false if (x = -2) and (y = 1): Although (-2 < 1), ((-2)^2 = 4) is not less than (1^2 = 1). Therefore, the predicate is false.

b. Provide different values from those in part (a) for which (Q(x, y)) is false: For example, if (x = -3) and (y = 2), the predicate is also false. Because while (-3 < 2), ((-3)^2 = 9) is not less than (2^2 = 4).

c. Explain why (Q(x, y)) is true if (x = 3) and (y = 8): Both conditions are met: (3 < 8) and (3^2 < 8^2). Therefore, the predicate is true.

d. Provide different values from those in part © for which (Q(x, y)) is true: For example, if (x = 4) and (y = 5), the predicate is also true. Because (4 < 5) and (4^2 < 5^2).

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1. If (x = 0.5), then (x = 0.5 < 1 / 0.5 = 2). So, the statement is false.
2. If (a = 2), then ((a – 1) / a = 1 / 2) which is a rational number, not an integer. So, the statement is false.
3. If (m = 1) and (n = 1), then (m n = 1 1 = 1 < 1 + 1 = 2). So, the statement is false.

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Description automatically generatedIf (x = 1) and (y = 1), then ( = + = 2). So, the statement is

Question 16: Rewrite each of the following statements in the form “∀ \_\_\_x, \_\_\_.”

a. All dinosaurs are extinct.

* ∀x, if x is a dinosaur, then x is extinct.

b. Every real number is positive, negative, or zero.

* ∀x, if x is a real number, then x is positive, negative, or zero.

c. No irrational numbers are integers.

* ∀x, if x is an irrational number, then x is not an integer.

d. No logicians are lazy.

* ∀x, if x is a logician, then x is not lazy.

e. The number 2,147,581,953 is not equal to the square of any integer.

* ∀x, if x is an integer, then x2 is not equal to 2,147,581,953.

f. The number −1 is not equal to the square of any real number.

* ∀x, if x is a real number, then x2 is not equal to -1.

Question 17: Rewrite each of the following in the form “∃ \_\_\_x such that \_\_\_.”

a. Some exercises have answers.

* ∃x, x is an exercise and x has an answer.

b. Some real numbers are rational.

∃x, x is a real number and x is rational

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a. ∃s ∈ D such that E(s) ∧ M(s).  
b. ∀s ∈ D, if C(s), then E(s).  
c. ∀s ∈ D, if C(s), then not E(s). (Or: ∀s ∈ D, C(s) → ~E(s), where ~ is not).  
d. ∃s ∈ D such that C(s) ∧ M(s).  
e. (∃s ∈ D such that C(s) ∧ E(s)) ∧ (∃s ∈ D such that C(s) ∧ ~E(s))

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a) False: Zero is not positive or negative.  
b) True: x has to be negative so Pos(- -x ) would equal a positive number.  
c) True: all integers are also real numbers.  
d) True: there exist real numbers that are not integers (Fractions).

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a)True, there exists a rectangle that is also a square.  
b)True, there exists a rectangle that is not a square.  
c)True, if its a square then it is a rectangle too.

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a) There exists an integer x such that x is prime and x is not odd.  
True. This is an existential statement which is defined to be true if and only if "x is prime and even" is true for at least 1 integer x. Since 2 is a prime number and it is not odd, this statement is true.  
b) If an integer x is prime, then x is not a perfect square.  
True.  
Since a prime number is an integer greater than 1 and is not a product of 2 similar positive integers. So a prime number cannot be a perfect square because if it were, it would be a product of 2 similar positive integers.  
c) There exists a x such that x is odd and x is a perfect square.  
True. This is an existential statement which is defined to be true if and only if "x is odd and x is a perfect square" is true for at least 1 integer x. Since 9 is a perfect square and it is odd, this statement is true.

Exercise 3.2

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a. There is a discrete mathematics student who is nonathletic.e. Some discrete mathematics students are nonathletic.  
The negation for "All discrete mathematics students are athletic" is simply that there exists at least one discrete mathematics student who is not athletic.  
Only the statements a. and e. are of this effect.

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Question 2

The negation of “All dogs are loyal” is that at least one dog is disloyal.  
Only the statements :  
c. Some dogs are disloyal.  
f. There is a dog that is disloyal.  
are identical to this.

Question 3

a.) ∃ a fish x such that x does not have gills  
b.) ∃ a computer c such that c does not have CPU  
c.) ∀ movies x,m is no more than 6 hours long  
d.) ∀ bands b,b has won less than or equal to 9 Grammy awards.

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Question 16: There exists a real number x such that x2≥1 and x≤0.

Question 17: ∃ an integer d, such that 6/d is an integer and d≠3.

Question 18: ∃ a real number x such that x(x + 1) > 0 and both x ≤ 0  
and x ≥ −1.

Question 19: ∃n∈Z such that n is prime and n is both even and n≠ 2.

Question 20: ∃ integers a, b, and c such that a − b is even and b − c is  
even and a − c is not even.

Question 21: ∃ an integer n, such that n is divisible by 6 and n is either not divisible by 2 or n is not divisible by 3.

Question 22: ∃ an integer x such that the square of x is odd and x is not odd.

Question 23: There exists a function that is differentiable and not continuous.

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Question 43: There exists a number that is divisible by 4 and not divisible by 8.

Question 44: There exists a person who is happy and does not have a large income.

Question 45: There exists a person who has a large income and is not happy.

Question 46: There exists a function that is a polynomial and does not have a real root.

Exercise 3.3

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a. n=16  
b. n=  
c. n=

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Question 5: The statement says that no matter what circle anyone might give you, you can find a square of the same color. This is true because the only circles are a, c, and b, and given a or c, which are blue, square j is also blue, and given b, which is gray, squares g and h are also gray.

Question 6: The statement says no matter what square anyone may give you, you can find a circle that is of a different color and above that square. This statement is true. Circle c is blue and above squares e, g, and h which are black or grey. Circle b is above square j which is blue. Hence for all squares x in the domain, there exists a circle y such that x and y have difference colors and y is above x.

Question 7: This is true because triangle d is above every square.

Question 8: This statement is true because there is a triangle f such that all the circles are above f.  
Also, triangle i is also a triangle such that all the circles are above i.  
There exists at least one triangle that satisfies the statement.

A close up of a math problem

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Question 13:

a. Statement: For every color, there is an animal of that color.  
There are animals of every color.  
b. Negation: ∃ a color C such that ∀ animals A, A is not colored C.  
For some color, there is no animal of that color.

Question 14:

Statement: There is a book that all people have read.  
Negation: There is no book that all people have read.

Question 15:

a. Statement: For every odd integer n, there is an integer k such that n = 2k + 1.  
Given any odd integer, there is another integer for which the given integer equals twice the other integer plus 1.  
Given any odd integer n, we can find another integer k so that n = 2k + 1.  
An odd integer is equal to twice some other integer plus 1.  
Every odd integer has the form 2k + 1 for some integer k.  
b. Negation: ∃ an odd integer n such that ∀ integers k, n ≠ 2k + 1.  
There is an odd integer that is not equal to 2k + 1 for any integer k.  
Some odd integer does not have the form 2k + 1 for any integer k.

Question 16:

a. There exists a real number u such that for all real numbers v, uv=v.  
There is a real number whose product with another real number is always equal to the other real number.  
b. negation: ∀ real numbers u, ∃ a real number v, such that uv ≠ v.  
For all real numbers u, there is a real number v, such that uv ≠ v.

Question 17:

a. Statement: For all rational numbers, there exists a pair of integers such that the rational number is the ratio of the two integers.  
For all rational numbers r, there exists integers a and b, such that r = a/b.  
b. Negation: ∃r∈Q, such that ∀a,b∈Z, r≠a/b.  
There exists at least one rational number r such that r is not equal to the ratio of a and b for any integers a and b.

Question 18:

a. Statement: For every real number x, there is a real number y such that x + y = 0.  
Given any real number x, there exists a real number y such that x + y = 0.  
Given any real number, we can find another real number (possibly the same) such that the sum of the given number plus the other number equals 0.  
Every real number can be added to some other real number (possibly itself) to obtain 0.  
b. Negation: ∃ a real number x such that ∀ real numbers y, x + y ≠ 0.  
There is a real number x for which there is no real number y with x + y ≠ 0.  
There is a real number x with the property that x + y ≠ 0 for any real number y.  
Some real number has the property that its sum with any other real number is nonzero.

Question 19:

a. Statement: there exists a real number x such that for all real numbers y, x+y=0.  
There is a real number that when that real number is added to any real number, the sum is zero.  
  
b. ∀x∈R, ∃y∈R such that x+y≠0.  
For every real number, there is another real number such that the sum of the two numbers is nonzero.  
For every real number x, there is a real number y, such that x+y≠0.

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Question 46:

a. True. Both triangles a and c lie above all the squares.  
b. Formal version: ∃x(Triangle(x) ∧ (∀y(Square(y) → Above(x, y))))  
c. Formal negation: ∀x(∼Triangle(x) ∨ (∃y (Square (y)∧ ∼Above(x, y))))

Question 47:

a. False. There is no triangle above circle b.  
b. Formal version: ∃x(Triangle(x) ∧ (∀y(Circle(y) → Above(x, y))))  
c. Formal negation: ∀x(∼Triangle(x) ∨ (∃y (Circle (y)∧ ∼Above(x, y))))

Question 48:

a. False. There is no square to the right of circle k.  
b. ∀x(Circle(x) → (∃y (Square(y) ∧ RightOf(y,x))))  
c. Formal negation: ∃x(Circle(x) ∧ (∀y(∼Square(y) ∨∼RightOf(y, x))))

Question 49:

a. True. For every object x, you can choose an object y that is not the same as x and has a different color than x.  
b. Formal: ∀x(Object(x) → (∃y (Object(y) ∧ x≠y ∧ DifferentColor(x,y)))  
c. Negation: ∃x (Object(x) ∧ (∀y (~Object(y) ∨ x=y ∨ ~DifferentColor(x,y))))

Question 50:

a. False. Take for instance object a. a ≠ c, but a and c have the same color.  
b. Formal: ∀x(Object(x) → (∃y (Object(y) ∧ (x≠y → DifferentColor(x,y))))  
c. Negation: ∃x (Object(x) ∧ (∀y ~(Object(y) ∨ (x≠y ∧ ~DifferentColor(x,y)))))

Question 51:

a. False. There is no object that has a different color from every other object.  
b. Formal version: ∃y(∀x(x = y → ∼SameColor(x, y)))  
c. Formal negation: ∀y(∃x(x = y ∧ SameColor(x, y)))

Question 52:

a. False. Every circle is not to the right of every triangle. For instance, circle b is not to the right of triangle c.  
b. Formal: ((∀x Circle(x) ∧∀y Triangle(y))→ RightOf(x, y))  
c. Negation: (((∃x Circle(x) ∨∃y Triangle(y)) ∧ ~RightOf(x, y)))

Question 53:

a. True. Circle b is black and squares h and j are black also.  
b. Formal version: ∃x(Circle(x) ∧ (∃y(Square(y) ∧ SameColor(x, y))))  
c. Formal negation: ∀x(∼Circle(x) ∨ (∀y(∼Square(y) ∨∼SameColor(x, y))))

Question 54:

a. False. All the triangles are gray. None of the circles are gray. Hence there are no triangles and circles that have the same color.  
b. Formal version: ∃x(Circle(x) ∧ (∃y(Triangle(y) ∧ SameColor(x, y))))  
c. Formal negation: ∀x(∼Circle(x) ∨ (∀y(∼Triangle(y) ∨ ∼SameColor(x, y))))

Exercise 3.4

A math problem with numbers and equations

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Question 2: The conclusion is: ∴ 0 is even.

Question 3: The conclusion: ∴ + =

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3) If an object is gray, then it is a circle.  
2) If an object is a circle, then it is to the right of all the blue objects.  
1) If an object is to the right of all the blue objects, then it is above all the triangles.

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Question 29

The reordering is:  
2. In contrapositive form: ∀x, if x is a square, then x is above all the black objects.  
3. ∀x, if x is above all the black objects, then x is to the right of all the triangles.  
1. ∀x, if x is to the right of all the triangles, then x is above all the circles.

Question 30

The reordering is:  
3. ∀x, if x is black, then x is a square.  
2. In contrapositive form: ∀x, if x is a square, then x is above all the gray objects.  
4. ∀x, if x is above all the gray objects, then x is above all the triangles.  
1. ∀x, if x is above all the triangles, then x is above all the blue objects.  
∴ if x is black, then x is above all the blue objects.